Álgebra (Grado en Ingeniería Informática)
Álgebra Lineal

Ejercicio 1.
Calcular, según los valores de $\alpha \in \mathbb{R}$, el rango de la matriz A_α:

$$A_\alpha = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha - 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 1 - \alpha \\ 0 & \alpha - 1 & 0 & 0 & 2 & 1 \\ 0 & -1 & 0 & 0 & 1 & 2 \end{pmatrix}$$

□

$$rg(A_\alpha) = rg \begin{pmatrix} 0 & (\alpha-2) & 0 & 0 \\ 2 & 0 & 1 & 1-\alpha \\ (\alpha-1) & 0 & 2 & 1 \\ -1 & 0 & 1 & 2 \end{pmatrix}$$

$$\begin{vmatrix} 0 & (\alpha-2) & 0 & 0 \\ 2 & 0 & 1 & (1-\alpha) \\ (\alpha-1) & 0 & 2 & 1 \\ -1 & 0 & 1 & 2 \end{vmatrix} = -(\alpha-2) \cdot \begin{vmatrix} 2 & 1 & (1-\alpha) \\ (\alpha-1) & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix}$$

$$= -(\alpha-2) \cdot \left[\underline{8} \ \underline{-1} + \underline{(\alpha-1)(1-\alpha)} + \underline{2(1-\alpha)} - 2\underline{(\alpha-1)} \ \underline{-2} \right]$$

$$= -(\alpha-2)\left[5 - \alpha^2 + 2\alpha - 1 + 4 - 4\alpha\right] =$$

$$= -(\alpha-2) \cdot \left[8 - 2\alpha - \alpha^2\right], \quad \alpha = \frac{2 \pm \sqrt{4+32}}{-2} =$$

$(*)$

$$= \frac{2 \pm 6}{-2} = \begin{cases} -4 \\ 2 \end{cases}$$

$$(*) = -(\alpha-2) \cdot \left[-(\alpha+4)(\alpha-2) \right] = (\alpha+4)(\alpha-2)^2 .$$

$$= 0 \iff \boxed{\begin{array}{c} \alpha = -4 \\ \vee \\ \alpha = 2 . \end{array}}$$

<u>Caso 1</u>. $\alpha \neq -4, 2$.

$$rg\, A_\alpha = 4 .$$

<u>Caso 2</u> : $\alpha = 2$.

$$rg\, A_2 = rg \underbrace{\begin{pmatrix} 2 & 1 & 1-\alpha \\ (\alpha-1) & \boxed{\begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix}} \\ -1 & & \end{pmatrix}}_{A}$$

$\det(A) = 0$, $\quad rg(A_\alpha) = 2$. porque $\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 \neq 0.$

<u>Caso 3</u> : $\alpha = -4$.

$$\operatorname{rg}(A_\alpha) = \operatorname{rg} \underbrace{\begin{pmatrix} 0 & -6 & 0 & 0 \\ 2 & 0 & 1 & 5 \\ -5 & 0 & 2 & 1 \\ -1 & 0 & 1 & 2 \end{pmatrix}}_{B}$$

$\det B = 0$, $\operatorname{rg}(A_\alpha) = 3$ porque

$$\det \begin{pmatrix} -6 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 2 & 1 \end{pmatrix} = -6 \cdot (-9) \neq 0.$$

$$\boxed{\begin{array}{l} \operatorname{rg}(A_\alpha) = 4 \quad \forall \alpha \neq 2, -4. \\ \operatorname{rg}(A_2) = 2 \\ \operatorname{rg}(A_{-4}) = 3 \end{array}}$$